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[20] R. H. Dicke, "The measurement of thermal radiation at microwave frequencies," *Rev. Sci. Instr.*, vol. 17, pp. 268–275; July, 1946. See especially p. 270.
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Some Considerations in the Design of Narrow-Band Waveguide Filters*

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***Summary*—It is natural that design considerations common to all kinds of filters should have received most attention in the literature. Some design considerations which are peculiar to waveguide filters (being due to the dispersive property of waveguides) are treated in this paper. It is shown that there are waveguide dimensions which minimize the filter dissipation loss and also the filter pulse power capacitance and which keep the nearest spurious-frequency response farthest from the fundamental pass band. Graphical data are presented to show how much is to be gained or lost by departing from the usual dimensions.**

Introduction

A FILTER (by definition) is required to operate selectively over a spectrum of frequencies. The specified frequency behavior can usually be met by a variety of possible designs, some of which are better suited than others to satisfy various additional requirements which may be specified or desirable; the electrical requirements may include low dissipation loss, high pulse power capacitance and freedom from spurious responses, in addition to the specified amplitude or phase characteristics.

Some general design considerations, common to lumped-constant, TEM-line (nondispersive) and waveguide (homogeneously dispersive) filters will first be summarized. Then we will discuss some special considerations that are peculiar to waveguide filters, which have an extra degree of freedom allowed by the choice of waveguide cutoff wavelength.

General Considerations for Band-Pass Filters

The design of narrow-band filters can be based on a lumped-constant, low-pass prototype circuit.[1] Formulas and tables[2] exist for filter elements g_i, which give maximally flat, Chebyshev and maximally flat time delay characteristics. Sometimes it is preferred to make the filter elements all equal to one another (all $g_i = g$). A microwave filter based on this prototype may be considered to consist of a cascade of identical cavities and will be called a matched-periodic (or just periodic) filter.[3,4] S. B. Cohn[5,6] has shown that such a filter has

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[4] L. Young, "Attenuation characteristics of periodic (equal-element) filters," PROC. IEEE, vol. 51, pp. 960–961; June, 1963.
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minimum dissipation loss[3,5,7] and maximum pulse power capacitance[6,8] for a given selectivity. It can further be shown that there is an optimum number n of resonators to give minimum midband dissipation loss in narrow-band filters.[9–11] When the stop-band attenuation is specified to be A db at any frequency in the stop band, then

$$n \approx 0.115A + 0.7 \tag{1}$$

resonators should be used.

In any filter there is a connection between the dissipation loss,[2,3,10] the equivalent power ratio[8,12] and the group delay, which are approximately proportional to one another. (The equivalent power ratio is inversely proportional to the pulse power capacitance.) All three quantities usually increase as the frequency moves away from midband, reach a fairly sharp peak just outside the pass band and then fall off quite rapidly.[8,12]

The properties briefly summarized above hold for waveguide as well as other types of filter. They show, for instance, how the pass-band dissipation loss can be minimized, or the pulse power can be maximized, in terms of the filter elements, or by controlling their number. It is our purpose now to show that for waveguide filters in particular, the waveguide dimensions can be optimized under various conditions. We shall be mostly concerned with the TE_{10} mode in rectangular waveguide and, to a lesser extent, with the TE_{01} mode in circular waveguide.

Midband Dissipation Loss

There exist several formulas for the dissipation loss in a transmission-line filter.[2,3,5,7,9–11,13] For narrow-band filters, which are reflectionless at midband and have only moderately high dissipation loss, a very useful formula has been given by Cohn.[2,5] Although his formula is exact only for vanishingly small dissipation loss, calculations by Taub[11] indicate that Cohn's formula holds quite accurately for up to about 3-db dissipation loss per resonator. It is also quite simple to extend Cohn's formula to where there is substantial reflection at midband,[2,3,7,10] and it then becomes

$$(\Delta L_A)_0 = (1 - |\rho_0|^2) \frac{4.343\omega_1'}{w} \sum_{i=1}^{n} \frac{g_i}{Q_{ui}} \text{ db}, \tag{2}$$

[7] L. Young, "Prediction of absorption loss in multilayer interference filters," *J. Opt. Soc. Am.*, vol. 52, pp. 753–761; July, 1962.

[8] G. L. Matthaei, L. Young, and E. M. T. Jones, *op. cit.*, ch. 15, sec. 15.03.

[9] S. Kh. Kogan, "Efficient design of band filters with small dissipative losses," in "Radio Engineering and Electronic Physics," vol. 7, pp. 1238–1244; English translation, Pergamon Press, London, England, August; 1963.

[10] L. Young, "Group delay and dissipation loss in transmission-line filters," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 215–217; May, 1963.

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[12] L. Young, "Peak internal fields in direct-coupled-cavity filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 612–616; November, 1960.

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where ρ_0 is the midband reflection coefficient, the g_i are the n elements of the low-pass prototype,[1,2] ω_1' is the cutoff frequency of the low-pass prototype, w is the fractional bandwidth of the band-pass filter and the Q_{ui} are the unloaded Q's of its n resonators. The g_i determine the shape of the characteristic, independently of the bandwidth, and one can normalize $\omega_1' = 1$ without loss of generality. Each resonator generally has the same unloaded Q; we shall consider only this case and therefore drop the numbering suffix i. For a matched filter ($|\rho_0| = 0$), (2) simplifies to

$$(\Delta L_A)_0 \propto \frac{1}{wQ_u}. \tag{3}$$

Thus to minimize the dissipation loss of any matched filter of given fractional bandwidth and given number of resonators, we have to maximize Q_u.

Consider a rectangular waveguide cavity of dimensions $a \times b \times c$, where the couplings are supposed to be small and are in the two walls $a \times b$ separated by a distance c (Fig. 1). This cavity forms part of a filter which is supposed to be narrow-band, with each cavity resonating in the TE_{101} mode, so that we can write to a good approximation

$$c = \frac{\lambda_{g0}}{2}, \tag{4}$$

where λ_{g0} is the guide wavelength at the design frequency f_0 given by

$$\frac{1}{\lambda_{g0}^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}, \tag{5}$$

and where λ_0 is the free-space wavelength at the design frequency f_0 and $\lambda_c = 2a$ is the cutoff wavelength.

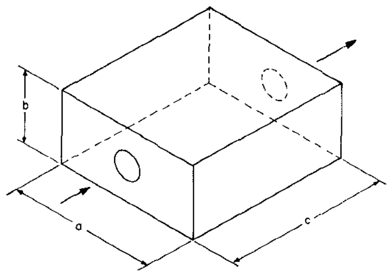

Fig. 1—Rectangular waveguide cavity.

b = Constant

We shall first consider the case where the waveguide height b is kept constant as the waveguide width a is varied. The ohmic losses in the walls of a given metal for a given current distribution are proportional to the surface area and are inversely proportional to the skin depth, δ. Since we are considering a *fixed frequency*, the skin depth is constant. Since we are stipulating a TE_{10} mode, the stored energy for a given current distribution is proportional to the volume. By definition Q_u is therefore proportional to the ratio of volume/surface area of the cavity and is maximum when that ratio is a maximum. (Compare with Moreno.[14]) For the TE_{10} mode, this reduces to the problem of maximizing the area of a rectangle $a \times c$ when the perimeter length $2(a+c)$ is given. The solution is a square, so that we find Q_u to be maximum when

$$a = c, \tag{6}$$

and so

$$\lambda_c = 2a = \lambda_{g0} = \lambda_0\sqrt{2} = 1.4142\lambda_0 \tag{7}$$

by (5).

[Parenthetically, it is worth pointing out that a similar argument can be applied to a TE_{01} mode in a circular waveguide cavity. Its ratio of volume/surface area is maximum when the cylinder height is equal to the cylinder diameter (this agrees with Moreno and Beringer[14,15]). Also, λ_c is equal to 0.82 times the diameter, and therefore Q_u is maximum and the dissipation loss of such a filter is minimum when λ_{g0} is twice the diameter and

$$\lambda_c = 0.41\lambda_{g0} = 1.08\lambda_0.] \tag{8}$$

Returning to the rectangular waveguide cavity (Fig. 1) with a TE_{101} resonance, its Q_u is given by[16]

$$Q_u = \frac{4}{\lambda_0^2 \delta}\left[\frac{b}{\left(\frac{a+2b}{a^3}\right)+\left(\frac{c+2b}{c^3}\right)}\right]$$

$$= \frac{1}{\delta}\left[\frac{1}{\frac{1}{b}+\frac{\lambda_0^2}{2}\left(\frac{1}{a^3}+\frac{1}{c^3}\right)}\right]. \tag{9}$$

We could again deduce (6) and (7) by differentiating (9) with respect to a, keeping the design wavelength λ_0, the skin depth δ and the waveguide height b constant, and using (4) and (5). Eq. (9) shows how the Q_u or the dissipation loss (3) of a filter varies with the waveguide cavity shape. This has been plotted in Fig. 2, Curve A, against λ_c/λ_0, for the case $b=\lambda_0/4$. (In Fig. 2, λ_c is the variable and λ_0 is considered constant.) The recommended limits of a waveguide band are typically at $\lambda_c/\lambda = 1.25$ and $\lambda_c/\lambda = 1.9$. ($\lambda_c/\lambda = 1$ is at cutoff, and the TE_{20} mode begins to propagate at $\lambda_c/\lambda = 2$.) Thus the optimum $\lambda_c/\lambda_0 = 1.414$, which gives minimum dissipation loss, lies well inside the usual waveguide band. However, the variation is very small (less than 3 per cent) over the entire band, as can be seen from Curve A, and so the choice of λ_c or a is not critical. In fact, it can readily be shown from (9) that Curve A in Fig. 2 (which holds for $b=\lambda_0/4$) always stays below 1.172 and attains

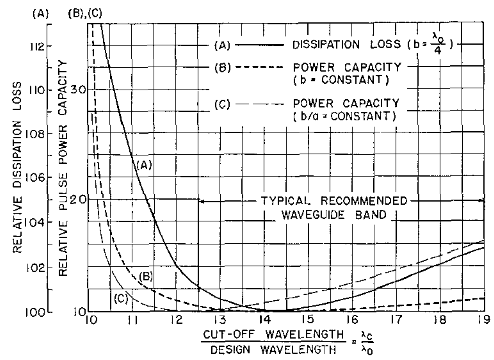

Fig. 2—Dissipation loss (A) and pulse power capacity (B and C) of rectangular-waveguide filters of specified design-frequency (λ_0=constant) and specified bandwidth, relative to their respective minimum values, as a function of the waveguide cutoff frequency. (Curve C also applies to TE modes in circular-waveguide filters.)

this value only in the two limits $\lambda_c = \lambda_0$ (cutoff) and $\lambda_c \to \infty$ (infinitely wide waveguide).

In general, when b is not necessarily equal to $\lambda_0/4$, the shape of the curve is similar to Curve A, Fig. 2, and the ratio of the dissipation loss at either end ($\lambda_c = \lambda_0$ or $\lambda_c \to \infty$) to the minimum dissipation loss, which still occurs at $\lambda_c = 1.414\ \lambda_0$, is $(4+\lambda_0/b)/(2\sqrt{2}+\lambda_0/b)$, which is always less than $\sqrt{2}$. For a single-mode waveguide b is less than $\lambda_0/2$, and the variation is less than 1.25. When $b=\lambda_0/4$, the expression reduces to $8/(2\sqrt{2}+4)=1.172$ as already mentioned. The variation becomes even less for smaller values of b.

b/a = Constant

So far, b has been kept constant. To compare two standard waveguides, the *ratio* b/a is approximately constant and usually equal or close to 0.5. In that case ($b/a=0.5$) Q_u increases monotonically from cutoff as a and b are increased together, finally reaching asymptotically a value twice that of Q_u at cutoff when a and b become infinitely large. (If it were plotted on Curve 2, the dissipation loss would decrease continually, reaching an asymptotic value of one-half of the dissipation loss at $\lambda_c/\lambda_0 = 1$.) It can be shown from (9) that for any constant value of b/a, the asymptotic value of Q_u to the value of Q_u at cutoff is $(a+2b)/2b$; this reduces to 2 when $b/a=0.5$. It can also be shown that there is no minimum in the curve of Dissipation Loss versus λ_c/λ_0 (compare Fig. 2) when b/a is constant and less than $1/3(\sqrt{2}-1)=0.806$ (for proof, see Appendix). When b/a is constant and greater than 0.806, there is a dissipation-loss minimum (Q_u maximum) which lies above $\lambda_c/\lambda_0=\sqrt{2}$. It moves closer to $\lambda_c/\lambda_0=\sqrt{2}$ as b/a increases, and for infinite b/a the minimum dissipation loss would be $1/\sqrt{2}$ times that at either end.

Since for the case of most interest ($b/a=0.5$) the dissipation loss versus λ_c/λ_0 curve has no minimum, this case is not plotted in Fig. 2. The smallest dissipation loss is the asymptotic value for infinitely large λ_c/λ_0. The relative dissipation loss with four important waveguide sizes is as follows: The dissipation loss at cutoff

[14] T. Moreno, "Microwave Transmission Design Data," Dover Publications, New York, N. Y., pp. 214–222; 1948.
[15] R. Beringer, "Technique of Microwave Measurements," C. G. Montgomery, Ed., M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 11, ch. 5, pp. 300–301; 1947.
[16] *Ibid.*, see ch. 5, p. 296.

($\lambda_c/\lambda_0=1$), at the low end of a waveguide band ($\lambda_c/\lambda_0 \approx 1.25$), at the high end of a waveguide band ($\lambda_c/\lambda_0 \approx 1.9$) and in the limit ($\lambda_c/\lambda_0 \rightarrow \infty$), all relative to the asymptotic value, are respectively 2.00, 1.52, 1.30 and 1.00 (when $b/a=0.5$). Therefore, as different standard waveguides are selected, the dissipation loss (or Q_u) varies by no more than about 17 per cent, so long as each filter is operated within the recommended band of its waveguide (λ_c/λ_0 between 1.25 and 1.9).

a = Constant

The dissipation loss always decreases (Q_u increases) when b is increased and a is kept constant. If we started with $a=c=2b$ and if b were then increased, the dissipation loss could be at most halved (Q_u could be at most doubled). If b were reduced towards zero, the dissipation loss would increase eventually in inverse proportion to b.

Pulse Power Capacitance

The pulse power capacitance, P, of a rectangular waveguide of cross section $a \times b$ (Fig. 1), propagating a traveling wave in the TE_{10} mode, is proportional to[8,17]

$$P \propto ab\left(\frac{\lambda_0}{\lambda_{g0}}\right). \tag{10}$$

When there is also a standing wave having a VSWR equal to S, then[8,12]

$$P \propto \frac{ab}{S}\left(\frac{\lambda_0}{\lambda_{g0}}\right). \tag{11}$$

It can readily be confirmed (Matthaei, Young and Jones[3] or Young[10,18]) that for any type of half-wave filter[18,19] (maximally flat or Chebyshev, etc.) of small fractional bandwidth w the internal VSWR S_i in the ith cavity is given by

$$S_i = \frac{2}{\pi}\,\frac{\omega_1'}{w}\left(\frac{\lambda_0}{\lambda_{g0}}\right)^2 g_i. \tag{12}$$

When the response shape, and hence the g_i, are given, (11) reduces to

$$P \propto wab\left(\frac{\lambda_{g0}}{\lambda_0}\right). \tag{13}$$

We shall differentiate this useful formula to obtain two different stationary points of P with respect to a, (1) for b=constant, and (2) for b/a=constant. In both cases we shall treat w and λ_0 as constants, since the bandwidth and the center frequency will have been specified at the outset. We shall also suppose that the number of resonators has previously been determined [for instance, by (1)].

b = Constant

Differentiating (13) for constant ω, λ_0 and b and using (5), yields after some manipulation

$$\lambda_c = 2a = \lambda_{g0} = \lambda_0\sqrt{2}, \tag{14}$$

which is the same as (7). However, while (7) or (14) represent an optimization for Q_u or the dissipation loss, it gives the least (*i.e.*, worst) pulse power capacitance. The variation of the pulse power capacitance as a function of λ_c/λ_0 is obtained from (5) and (13) and is plotted in Fig. 2, Curve B. (To repeat: In Fig. 2, λ_c is the variable and λ_0 is considered constant.) It is seen that the variation over the recommended waveguide band is 12 per cent. However, considerable improvement in the pulse power capacitance may be obtained by reducing the guide width a to near cutoff. Theoretically, the curve would go to infinity if the filter bandwidth could be made infinitesimally small; this result is quite different from that for Curve A, which always stays below 1.172 (for $b=\lambda_0/4$). Moreover, it is well known[17] that the pulse power capacitance of waveguide approaches zero as the guide width a is reduced to cutoff. This can be seen from (10) when $\lambda_{g0} \rightarrow \infty$. The explanation of the apparent paradox (the filter tending towards infinite pulse power capacitance when the waveguide pulse power capacitance goes to zero) is that as $\lambda_c=2a$ is reduced towards cutoff, the guide becomes more dispersive, and any fixed (frequency) fractional bandwidth w corresponds to an increasing fractional bandwidth in reciprocal-guide wavelength w_{λ_g} which for small bandwidths is given by

$$w_{\lambda_g} = w\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2. \tag{15}$$

[This shows up as a factor in (12).] As the guide width approaches cutoff, the external couplings therefore have to increase to maintain constant bandwidth. This lowers each internal VSWR S_i more than it lowers the pulse power capacitance of the waveguide under traveling-wave conditions. Therefore the final formula, (13), shows that the midband pulse power capacitance near waveguide cutoff is nearly proportional to the guide wavelength λ_{g0} which increases without limit. This explains the perhaps unexpected, unbounded increase in pulse power capacitance of a waveguide *filter of given bandwidth* operating near waveguide cutoff.

Before building a filter operating near waveguide cutoff, the following considerations should be taken into account:

Firstly, the dissipation loss increases. For example, it is seen from Fig. 2, Curves A and B, that when the pulse

[17] R. M. Walker, "Microwave Transmission Circuits," G. L. Ragan, Ed., M.I.T. Rad. Lab. Ser., vol. 9, McGraw-Hill Book Co., Inc., New York, N. Y., pp. 190–193; 1948.

[18] L. Young, "Stepped impedance transformers and filter prototypes," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 339–359; September, 1962.

[19] L. Young, "Direct-coupled-cavity filters for wide and narrow bandwidths," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 162–178; May, 1963.

power is tripled, the dissipation loss increases by about 15 per cent for the case $b=\lambda_0/4$. (As mentioned before, the dissipation loss increases by at most 17.2 per cent, in that case.)

Secondly, λ_c/λ_0 is as low as 1.015 where the pulse power capacitance is tripled. This means in practice that the filter fractional bandwidth will have to be less than about 1 per cent. It also means that the a dimension of the waveguide has to be maintained very accurately. If one tried to increase the pulse power capacitance indefinitely, the filter bandwidth would have to shrink to zero.

Thirdly, the guide wavelength becomes large (for example, $\lambda_{g0}\approx 6\lambda_0$ where the pulse power capacitance is tripled), so that the filter becomes long; it is even longer when one includes the inhomogeneous transformer[20,21] or long taper which would probably be required to match to standard waveguide at each end. It may therefore be more practical to build more cavities or longer cavities (multiples of $\lambda_{g0}/2$ long) in standard waveguide ($\lambda_c\approx 1.5\ \lambda_0$) to increase the pulse power capacitance.

Fourthly, as the guide becomes more dispersive, the nearest spurious frequency moves closer to the fundamental pass band. It would therefore be better to build more less-dispersive cavities in standard waveguide, as already suggested above.

So far it has been supposed that the number of resonators n had been predetermined. Suppose, instead, that we are limited in over-all length; for instance, the filter may have to be fitted into a length L. When we try to optimize a ($L=$constant), we find that a should be infinite. Thus, to optimize the pulse power capacitance when the number of resonators n is given, the filter *length* would increase without limit; while if the filter length, L, were specified, the filter *width* would increase without limit. One practical limitation in both cases is the existence of higher-order modes, and this will be discussed in the next section. Before that, however, we shall consider the case $b/a=$constant, rather than $b=$constant, as we also did in connection with the dissipation loss.

$b/a=$ *Constant*

As pointed out in a previous section, keeping $b/a=$constant corresponds to the case of selecting one of a number of possible standard waveguides. Rewrite (13) as follows:

$$P \propto w\left(\frac{b}{a}\right)a^2\left(\frac{\lambda_{g0}}{\lambda_0}\right) \tag{16}$$

and now treat w, λ_0 and b/a as constants. Using (5) and differentiating, now leads to

$$\lambda_c = 2a = \frac{\lambda_{g0}}{\sqrt{2}} = \lambda_0\sqrt{\frac{3}{2}} = 1.2247\lambda_0 \tag{17}$$

instead of (14). Again this gives the least pulse power capacitance; its variation as a function of λ_c/λ_0 is obtained from (5) and (16) and is plotted in Fig. 2, Curve C. If we wished to stay inside the recommended waveguide band, then Curve C shows that for the greatest pulse power capacitance a waveguide should be selected so that the filter pass band lies at the high-frequency end of the recommended waveguide band. The minimum pulse power capacitance occurs when the filter pass band lies just below the low-frequency end of the waveguide band; it is seen from Curve C that the improvement in pulse power capacitance can be as much as 63 per cent. On the other hand, the pulse power capacitance increases without limit as the guide approaches cutoff, so that, instead of picking the largest waveguide which contains the filter pass band in its recommended band, one could select the smallest waveguide which is just not cutoff, and apparently fare even better. However, there are several disadvantages to this second alternative, which have already been enumerated above.

Curve C can also be used for the TE_{01} mode in a circular-waveguide filter[8] and generally holds for any TE mode in a waveguide filter of fixed cross-sectional shape,[22] of which rectangular waveguide with fixed b/a ratio and circular waveguide, are the two most important examples.

$a=$ *Constant*

When a is constant the pulse power capacitance is directly proportional to b.

Spurious Responses (Higher-Order Modes)

The first spurious resonance in the rectangular waveguide filter (Fig. 1) will generally occur at a frequency just above that for which

$$\lambda_0 = a \quad \text{or} \quad c, \tag{18}$$

whichever is the larger (unless special precautions are taken to suppress the resonance). It follows from (18) that the nearest spurious response is the farthest away when

$$a = c, \tag{19}$$

[20] L. Young, "Inhomogeneous quarter-wave transformers," *Microwave J.*, vol. 5, pp. 84–89; February, 1962.

[21] G. L. Matthaei, L. Young, and E. M. T. Jones, *op. cit.*, see ch. 6, sec. 6.11 and 6.12.

[22] Since the cross-sectional area is then proportional to λ_c^2, which replaces a^2 in (16). The (λ_0/λ_{g0}) dependence in (11) is true for all TE modes. Compare Moreno[14], p. 124.

and so

$$\lambda_c = 2a = \lambda_{g0} = \lambda_0\sqrt{2}, \tag{20}$$

which is the same as (7) and (14). Thus, in a rectangular-waveguide filter, there is always the possibility of a spurious resonance within about 41.4 per cent of the fundamental pass band.

Conclusions

When the filter response is already determined (its operating frequency, fractional bandwidth w and number of resonators n being given), then in rectangular waveguide of *constant height* b the minimum dissipation loss (maximum unloaded Q), the minimum pulse power capacitance and the greatest separation from the nearest spurious resonance, are obtained with cavities that are square (length, c = width, a), corresponding to (7), (14) or (20), showing that $\lambda_c/\lambda_0 = \sqrt{2}$. Unless pulse power capacitance is an over-riding consideration, this is the best place to operate. The advantages of deviating appreciably from the square shape ($c = a$) when pulse power capacitance is critical may be more apparent than real; it may be better to redesign the filter, adding more cavities to allow for looser couplings. Graphs were given (Curves A and B, Fig. 2) showing how much is to be gained or lost by departing from the minima determined by (7) and (14). In the case of the dissipation loss in a single-mode, rectangular, waveguide filter of constant height, it was found that varying the waveguide width does not increase the dissipation loss by more than 25 per cent above its minimum value (which occurs when $c = a$).

When the *aspect ratio* b/a is kept *constant* and less than 0.806, the dissipation loss of the filter decreases continually as the waveguide size is increased. The minimum pulse power capacitance of the filter, when the aspect ratio b/a is kept constant, occurs at $\lambda_c/\lambda_0 = \sqrt{3/2}$, by (17), which is just below the low-frequency end of the usually recommended waveguide band. The pulse power capacitance increases in both directions (Fig. 2, Curve C), rising in one direction toward the high-frequency end of the recommended band, but also rising in the other direction towards cutoff. Curve C, Fig. 2, also holds for all TE modes in waveguides with constant cross-sectional shape, including all TE modes in circular waveguide.

Appendix

Dissipation Loss for b/a = Constant

Let

$$x = \lambda_c/\lambda \quad \text{and} \quad y = \lambda_g/\lambda; \tag{21}$$

then (5) reduces to

$$\frac{1}{x^2} + \frac{1}{y^2} = 1. \tag{22}$$

Let

$$b/a = \text{constant} = 1/2K; \tag{23}$$

then (9) reduces to

$$\frac{1}{Q_u} \propto \frac{K}{x} + \frac{1}{x^3} + \frac{1}{y^3} \cdot \tag{24}$$

For a stationary point, differentiate (24) and use (22). This gives

$$K^2x^4 + (6K - 9)x^2 + 18 = 0. \tag{25}$$

For the stationary point to be real, we require

$$(6K - 9)^2 - 72K^2 \geq 0, \tag{26}$$

which reduces to

$$b/a = 1/2K \leq \frac{1}{3(\sqrt{2} - 1)} \cdot \tag{27}$$

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